MATHEMATICAL MODELING OF ANATOMICAL STRUCTURES BY MEANS OF SPHERICAL HARMONICS

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ABSTRACT

The mathematical description of moving anatomical structures is of increasing importance when dealing with tumor irradiation by particle beams. A mathematical description of the orientation of anatomical structures by principal inertia axes and of the shape by spherical harmonics is given. Structures of convex and concave shape are decomposed and reconstructed with index numbers L_{max} = 10. Conformity of decomposed and reconstructed shape is measured by a similarity index. For irradiation therapy the influence of a security margin is considered. No significant improvement of conformity was found by using different coordinate systems for decomposition and reconstruction.

Index Terms - Spherical harmonics, tumor movement, lung tumor, inertia tensor, similarity index, 4D-CT data.

1. INTRODUCTION

For high precision radio therapy of moving targets the possibility of mathematical models gains increasing importance. Segmentation of structures will be derived from data taken by the new imaging technology of 4D-CT. These data allow, by manually delineating the structures, the creation of time dependant contour models, which have to be converted into mathematical models. The underlying mathematical background of this work is the need for highly exact conformal irradiation of moving targets. Using a cyber-knife [1], or the newly developed particle-scan-beam method, allows the aligning of the radiation field to the moving target, if a motional model exists [2]. Currently the target motion is considered by modern photon therapy in increasing the irradiation field margin.

The model has to describe the time dependant position, orientation and shape. As a mathematical frame work spherical harmonics (SH) are used. SH are already applied in static modeling of brain tumors [3] and for the segmentation of poorly visible structured organs [4]. Also changes of shapes, separately in long time intervals taken images, are described by SH [5]. They are also used in connection with 4D-CT in modeling lung movement [6]. Aim of the presented paper is to investigate the necessary index number to decompose and to reconstruct anatomical structures with respect to the chosen coordinate system. The position of the center of mass, the orientation of the principal inertia axes and the coefficients of the SH describe the mathematical model of moving anatomical structures.

2. MATERIALS AND METHODS

The application of SH was tested on time independent 3D-CT and on time dependent 4D-CT data sets. The target volumes concerned were delineated within the slices by a radiologist. The contours, consisting of N defined data points, were triangulated by means of cubic B-splines as 3D volumes. The coordinates of the volume are defined with respect to the coordinate system of the CT data set. The center of mass of the target volume defines its position within this coordinate system. Its inertia tensor I with respect to a parallel coordinate system with origin in the center of mass is calculated. I is defined as

$$\mathrm{I} = \begin{bmatrix} I_{\mathrm{xx}} & I_{\mathrm{xy}} & I_{\mathrm{xz}} \\ I_{\mathrm{yx}} & I_{yy} & I_{\mathrm{yz}} \\ I_{\mathrm{zx}} & I_{\mathrm{zy}} & I_{\mathrm{zz}} \end{bmatrix}, \text{ in which are}$$

$$I_{\mathrm{i,j}} = \sum_{\substack{\text{for all voxels} \\ \text{of volume}}} \Delta \mathrm{m} * (\mathrm{r}^2 \delta_{\mathrm{ij}} - \mathrm{r_i r_j})$$

$$\delta_{\mathrm{i,j}} = \begin{cases} 1 \text{ for i} = \text{j} \\ 0 \text{ for i} \neq \text{J} \end{cases}$$

and r the radius vector.

Δm describes the weight coefficients of the target voxels. $\Delta m = 1$ if anatomic structures of homogeneous mass distribution can be assumed. The eigenvalues u_k of matrix I define a rotational matrix $U = (u_1, u_2, u_3)$ with $\det|U| = 1$, which diagonalizes the inertia tensor I :

$$I_{diag} = U^{-1}\, I\, U.$$

U defines the angles α, β and γ with respect to the axes x, y and z, which project the volume concerned onto the principal inertia axes. They define the orientation of the volume with respect to the CT coordinate system.

The target surface may now be decomposed into SH. They form a complete and orthonormal set of eigenfunctions $Y_{L,m}$. The given data points $P_i = P(x_i, y_i, z_i) = P(r_i)$, $i = 1, \ldots, N$, of the surface may be developed as a sum of SH with $(L_{max} + 1)^2$ summands :

$$P_i = \sum_{L=0}^{L_{max}} \sum_{m=-L}^{+L} C_{L,m}\, Y_{L,m}(\Omega_i)\,, \qquad \text{eq.(1)}$$

whereas $L = 0,\ldots, L_{max}$ and $m = -L,\ldots, +L$. The coefficients $C_{L,m}$ in eq.(1) represent the shape of the structure. These unknown coefficients are found by minimizing the sum of the squared distances Δ from the defined N points P_i of the given surface :

$$\Delta = \sum_{i=1}^{N} \left(\sum_{L=0}^{L_{max}} \sum_{m=-L}^{+L} C_{L,m}\, Y_{L,m}(\Omega_i) - P_i \right)^2 \rightarrow \text{minimum}$$

The minimum is got by solving the differential equation

$$\frac{\partial \Delta}{\partial C_{L,m}} = 0\,.$$

$$\Rightarrow \sum_{i=1}^{N} \left(\sum_{p=0}^{L_{max}} \sum_{q=-p}^{+p} C_{p,q}\, Y_{p,q}(\Omega_i) - P_i \right) Y_{L,m}(\Omega_i) = 0$$

with solution

$$\sum_{p=0}^{L_{max}} \sum_{q=-p}^{p} C_{p,q} \left(\sum_{i=1}^{N} Y_{p,q}(\Omega_i)\, Y_{L,m}(\Omega_i) \right) = \sum_{i=1}^{N} P_i\, Y_{L,m}(\Omega_i) \qquad \text{eq.(2)}$$

Eq.(2) represents a linear equation system of type

$$A * \mathbf{C_{L,m}} = \mathbf{P}$$

composed of $(L_{max}+1)^2$ equations and $(L_{max}+1)^2$ unknown coefficients $C_{L,m}$. To find good solution conditions for the number N of the data points, the condition

$$N >> (L_{max} + 1)^2 \qquad \text{eq.(3)}$$

should hold. The values of $C_{L,m}$ define the shape of the anatomical structure. After calculating the coefficients $C_{L,m}$, the shape of the structure may be reconstructed according eq.(1) using the $Y_{L,m}$ for given L_{max} :

$$R_i = \sum_{L=0}^{L_{max}} \sum_{m=-L}^{+L} C_{L,m}\, Y_{L,m}(\Omega_i)\,, \qquad \text{eq.(4)}$$

in which the R_i represent the new radii of the N data points of the reconstructed surface.

The new data points are triangulated and the new volume is calculated. [7,8]. As a measure of the conformity of the reconstructed volume V_{rec} with the original volume V_{orig} of the anatomical structure a similarity index

$$SI = \frac{V_{rec} \cap V_{orig}}{V_{rec} \cup V_{orig}} \qquad \text{eq.(5)}$$

is used.

We investigated the decomposition and reconstruction of the target volumes according to eq.(4) using coordinate systems with axes parallel to the data cube and origin within the center of mass of the target volume and in the coordinate system of inertia as well. We decomposed and reconstructed anatomical structures in dependence on index number L_{max}. Comparison of comformity was done using the size of the volumes in units of voxels and the similarity index according eq.(5) as well. The intention is to gain knowledge of the appropriate value of the index number.

3. MEASUREMENTS

The mathematical framework was first tested with two different models of stationary anatomical structures. The first one was a cervix which showed a significant concave shape, the second was a prostate with a mainly convex shaped structure. The CT data sets, which were given in the DICOM data format, were linearly interpolated to have the same geometric resolution in all three dimensions. The structures were interactively delineated and triangulated. Decomposition and reconstruction of the surface of the cervix was done with respect to two different coordinate systems, one of which was parallel to the data set coordinates; the other one was defined by the principal inertia axes. A comparison of both volumes in units of voxels and a computation of the similarity indices SI was made to investigate potential advantages for faster shape reconstruction with lower index numbers. (table 1). In the case of the convex shaped prostate no such advantage was expected.

Subsequently we applied the algorithm to a convex shaped, moving lung tumor extracted from a 4D-CT, in which 12 respiratory phases, six inspiration and six expiration phases each were registered. All 12 3D-images were extracted, decomposed and reconstructed.

4. RESULTS AND DISCUSSION

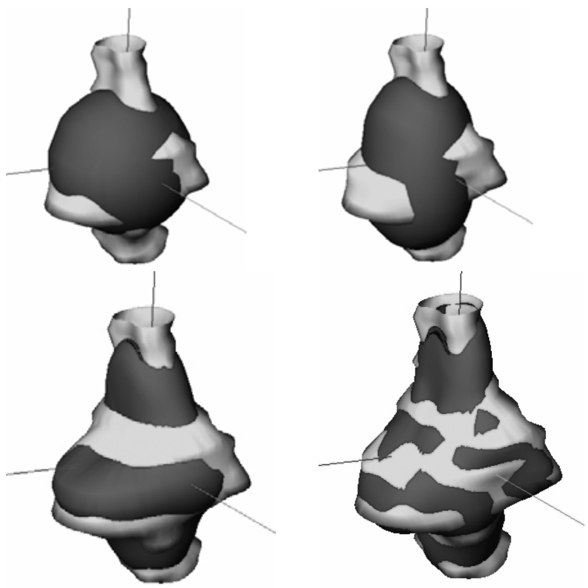

Fig. 1 : Development and superposition of the reconstructed volume (dark grey) onto the original volume of the cervix (light grey) for different values of index number L_{max} = 0, 2, 4 and 8 (from left to right and from top to bottom)

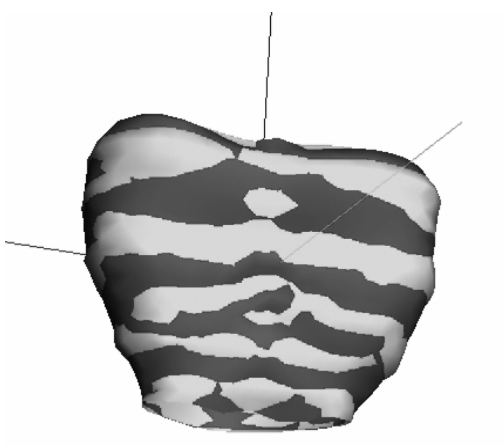

Fig. 2 : Superposition of the reconstructed volume of a prostate (dark grey) onto the original volume (light grey) for index number L_{max} = 10.

The shape of the cervix (fig. 1) was defined within 39 slices of the given DICOM data set. By interpolation 1394 points within 46 slices were chosen and triangulated with 2717 triangles. The approach of the reconstructed shape to the original shape, defined by the similarity index, was already better than 85 % with index number L_{max} = 10 as shown in Table 1.

The measurements of SI in table 1 show the approach of the reconstructed volume to the original volume when using index numbers in the range of L_{max} = 0 to 10. The mathematical model of tumors for the purpose of irradiation gives sufficient accuracy considering a margin of safety for conformity when using photons.

In the case of the prostate (fig. 2) the volume was defined by 256 787 voxels with an edge length of 0.975 mm. Delineation was done using 964 data points and triangulation with 1862 triangles. The fraction of the original volume, not covered by the reconstructed volume with L_{max} = 10, added up to 4 987 voxels or 1.9 % of the original volume, resulting in a similarity index of SI = 95.8 %. Increasing L_{max} = 20 the respective number of voxels was 1691 or 0.65 % of the original volume, resulting in a similarity index of SI = 98.6 %. After implementation of a security margin of 1 mm for irradiation planning, the not covered volume was 0.27 % for L_{max} = 10 respectively 0.1 % for L_{max} = 20. In this case the measure of a similarity index was not applicable.

Until now no significant improvement of conformity was found when the anatomic structures were decomposed into SH and reconstructed with respect to the principal inertia axes.

	Reconstructed volume in parallel coordinate system	Reconstructed volume in principal inertia coordinate system
L_{max}	Similarity Index SI in %	Similarity Index SI in %
0	53,69	52,45
1	52,61	52,00
2	56,57	56,81
3	62,73	62,46
4	72,96	73,12
5	77,30	77,21
6	80,14	80,07
7	80,79	80,19
8	84,30	83,33
9	84,00	83,77
10	85,44	84,86

Table 1 : Development of SI in dependence on index number L_{max}.

Increasing the index number L_{max} from 10 up to 60 needs much more given data points to fulfill eq.(3). Besides this the computation time increases considerably as shown in table 2.

L_{max}	Computation time
10	9 s
20	50 s
30	2.8 min
40	7.3 min
50	18.1 min
60	40,7 min

Table 2 : Computation time in dependence on L_{max}.

The spherical shaped lung tumors with diameter of approx. 1.5 cm were extracted from the 4D-CT. They were defined by voxels with size of 0.79 mm in x- and y-direction respectively and 3.00 mm in z-direction and thus showed a low geometrical resolution. Delineation was done with approx. 38 data points in 7 slices each, resulting in 211 triangles.

To allow interpolation between the tumor shapes of the discrete respiratory phases, the 3D wire mesh, generated by the delineation data points, was sampled with mutually corresponding data points and decomposed in SH with index number L_{max} = 10. Superposition of low resolution delineated lung tumors with high resolution reconstructed models resulted in similarity indices of about 90% depending on local shape.

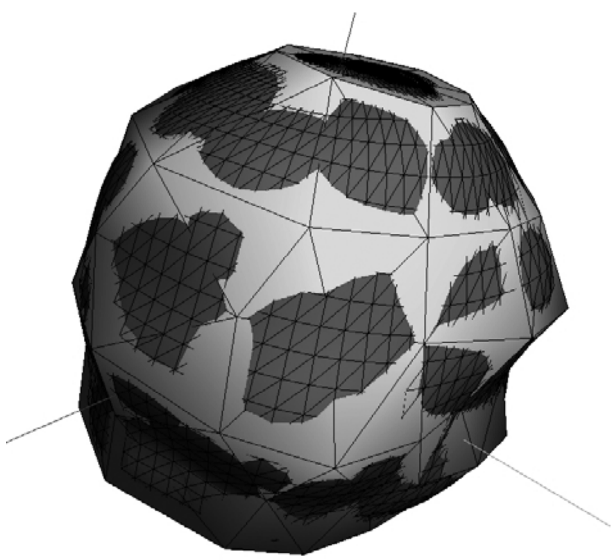

Fig. 3 : Superposition of a reconstructed volume of a lung tumor (dark grey) onto the original volume (light grey) for index number L_{max} = 10.

The discrete positions of moving tumors described by the center of mass, the orientation of non-spherical shapes described by the principal inertia axes and the shape described by spherical harmonics seems to be an appropriate mathematical model for moving anatomical structures.

Two different measures of the conformity of original and reconstructed tumors were used : a similarity index and the number of exceeding voxels. However, for irradiation planning with a security margin the similarity index is not applicable. For decomposition and reconstruction of tumors with spherical harmonics index number of 10 to 20 are sufficient, provided there are no filamentous extensions of tissue.

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